Defect production behind the shock wave front of an inhomogeneous quench

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The creation of defects behind the half kink in the presence of external force distributions is considered. The influence of external potentials on kink production behind the shock wave front of an inhomogeneous quench is examined. It is shown that depending on the impurity strength and orientation its interaction with the front of the decaying false vacuum, even in homogenous systems, may lead to single or even multiple defect production in the vicinity of the impurity center.

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I. INTRODUCTION

Presently, symmetry-breaking phase transitions are ubiquitous in the description of condensed matter systems, elementary particle physics, and cosmology. A uniform background for the description of continuous phase transitions has been proposed by Kibble and Zurek in their illuminating articles [1] and tested numerically in a series of papers [2]. The validity of this scenario was also confirmed experimentally in He^3 [3]. According to the Kibble-Zurek scenario, during the transition to the broken-symmetry phase, the order parameter in causally separated regions of space chose different vacuum values. The coexistence of different domains of irreconcilable low-temperature vacua on boundaries leads to the creation of topological defects. The initial density of the defects produced follows from the observation that if the system evolves towards the critical point as a consequence of critical slowing down, the relaxation time diverges and perturbations of the order parameter propagate very slowly. If the time of propagation of density perturbations over the correlated regions becomes comparable with the relaxation time, the field configuration in the system freezes in. The same time after transition the system regains capacity to respond for the change of external parameters. The correlation length at the instant of freeze out sets the size of the regions over which the same vacuum can be selected. Hence, it sets the resulting density of the topological defects. The correlation length at that instant describes the size of the defect and therefore the density of defects is limited by their size at the time of freeze-out.

This description concerns the homogenous quench in an ideally homogenous infinite system with no boundaries. In real systems a phase transition is often associated with a large degree of inhomogeneity. Examples of generically inhomogeneous systems are superconducting layers and liquid crystals. Actually, in liquid crystals one could observe transitions that can only approximately be considered as transitions of the second type. On the other hand, superfluid liquids in nature are free of impurities; however, contamination of superfluid helium can also be achieved by application of some artificial method such as the aerogel technique [4]. Considerations of the impact coming from the impurities on the production of defects during continuous transition were performed in one and more dimensions [5]. It was proved that the number density of the produced defects is characterized not only by the correlation length at the instant of freeze out but also by the length scale describing the distribution of impurities in the system. This situation was described in Ref. [6]. It seems that in an inhomogeneous medium there are two components of the number density of produced defects. One part of the defects is produced in the neighborhood of the impurities and seems to be persistently confined by the impurities. The second component consists of free defects produced in homogenous areas located between impurities. This component decreases with time as a consequence of the kink-antikink annihilation. The creation of kink-antikink pairs is also possible, but at this stage of evolution it is much less efficient than annihilation. For late times, i.e., when the system is in the stationary state, the probability of the creation of pairs is determined by the proper Boltzmann exponent [7]. The contribution of each component to the total number density depends on the separation and strength of the impurities. For instance, if the separation of the strong impurities is comparable with correlation length (or smaller) then there is no room for creation of free defects. In the opposite regime, i.e., for widely separated impurities, the main part of the kinks is produced in areas located between impurity centers and therefore the total number density is dominated by free kinks. These two components are also visible at late times when the system became stationary [8].

In the real condensed matter systems, we have to face two types of complications. First is inhomogeneity of the medium, and second is inhomogeneity of the quench. Inhomogeneity of the quench is a consequence of the fact that the change of thermodynamic parameters is unlikely to be ideally uniform. As a consequence of this inhomogeneity the symmetry is initially broken in some region of the system. The order parameter chose some vacuum value in this region and when the broken symmetry region grew, the choice of the vacuum value was to some degree enforced in the neighborhood of the initial region. This mechanism of inhomogeneous quench in a homogenous medium could lead to suppressing of or even halting of production of topological defects [9]

In this paper we consider the influence of the impurities on creation of kinks during an inhomogeneous quench. In the next section we illustrate mechanisms of the formation of defects behind the front of the decaying false vacuum in the presence of impurities. The last part contains some remarks.

II. CREATION OF KINKS BEHIND THE SHOCK WAVE FRONT

Let us consider the dissipative ϕ^4 model in one spatial dimension

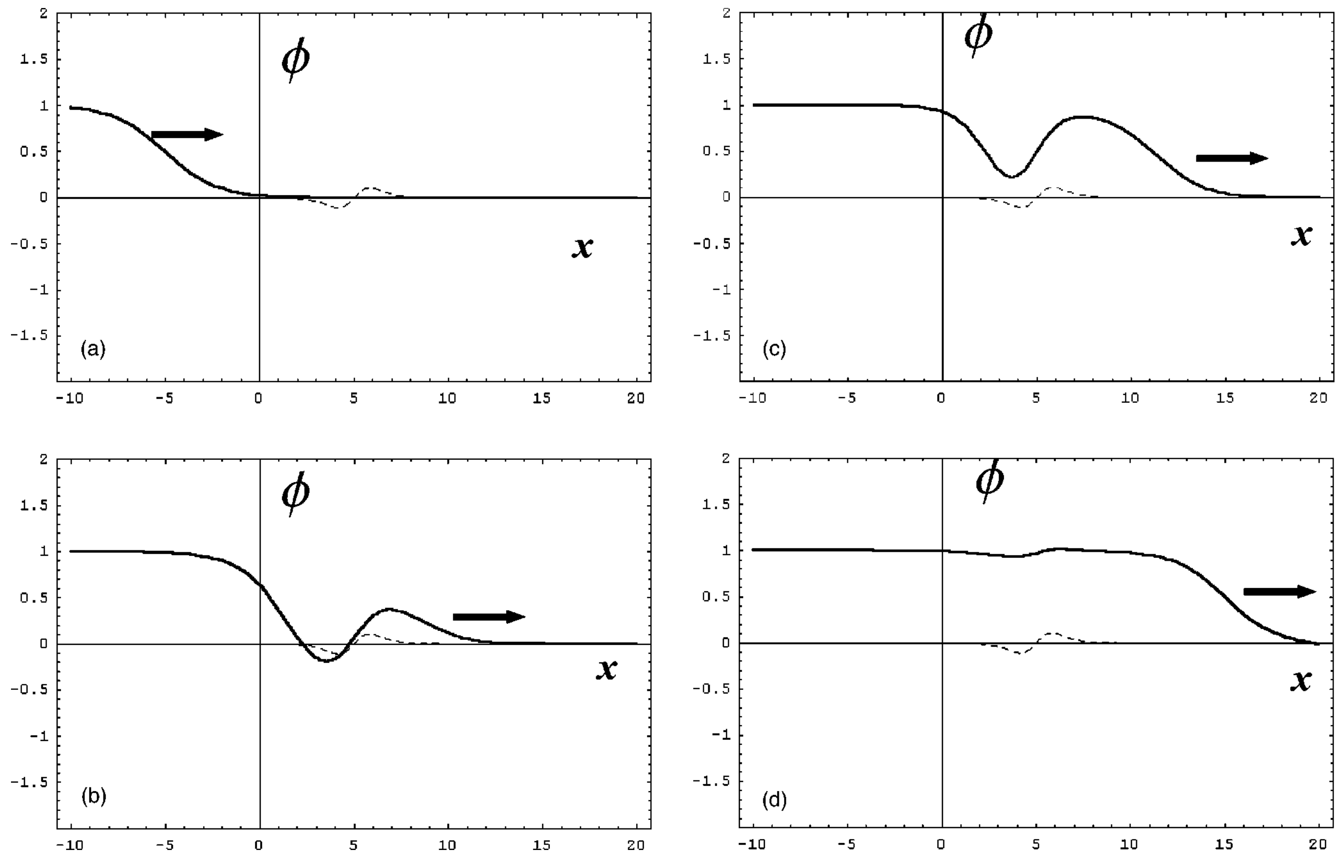

FIG. 1. Initially the half kink moves in the direction of the impurity (a). The modulus of the amplitude of the force distribution $|\mathcal{A}|$ is small and therefore after a period of interaction of the half kink with the impurity center (b),(c) we obtain a free half kink moving in the initial direction (d). The impurity is too weak to produce any additional kink structures in its vicinity. The parameters in the plots are the following: $\mathcal{A}=0.28$, $\Gamma=10$, $\lambda=1$, $a=1$, $x_0=5$. The dashed line represents the impurity force distribution.

$$\frac{1}{c^2}\partial_t^2\phi(t,x)+\Gamma\partial_t\phi(t,x)=\partial_x^2\phi(t,x)+a\phi(t,x)-\lambda\phi^3(t,x), \tag{1}$$

where ϕ is a real scalar field and λ, Γ, c, a are positive constants. For positive a the system remains in the broken symmetry phase. We know that in these settings the equation of motion posesses a stationary solution, called a half kink, which describes a decay of the false vacuum

$$\phi_H^{(\pm)}(x-v_st)=\pm\sqrt{\frac{a}{\lambda}}\frac{1}{1+e^{\sqrt{(a/2)}\gamma(x-v_st)}}, \tag{2}$$

where $\gamma=1/\sqrt{1-v_s^2/c^2}$ and $v_s=\pm c/\sqrt{1+2\Gamma^2c^2/9a}$. The velocity of the half kink is determined in a natural way by the equilibration of the forces coming from the potential and the friction present in the system. The solution described above exists for Eq. (1) which contains the inertia term and also in the overdamped model achieved in the large c limit.

If we enrich Eq. (1) by adding a noise $\eta(t,x)$ term and assume space and time dependence of the quantity $a(t,x)$ which allows for inhomogeneous change of the sign of a, than we obtain an equation modeling the inhomogeneous phase transition from the symmetric to broken symmetry phase

$$\frac{1}{c^2}\partial_t^2\phi(t,x)+\Gamma\partial_t\phi(t,x) =\partial_x^2\phi(t,x)+a\phi(t,x)-\lambda\phi^3(t,x)+\eta(t,x). \tag{3}$$

It is usually assumed that η is the Gaussian white noise of temperature T:

$$\langle\eta(t,x)\rangle=0,$$

$$\langle\eta(t,x)\eta(t',x')\rangle=2\pi\Gamma kT\delta(x-x')\delta(t-t'). \tag{4}$$

In Ref. [9] some particular forms of the function $a=a(t-x/v)$ have been considered. One choice was the step function $a(t,x)=a_0\mathrm{sgn}(t-x/v)$ and another was the oblique step function. Both choices describe the phase front which arises as a consequence of the nonuniform change of thermodynamic parameters. The authors, who consider inhomogeneous quench in a homogenous medium, analyze the stability of the half-kink solution in the presence of the pressure shock wave. They compare the velocity of the phase front with the speed of the interface between decaying false and true vacua. They conclude that if the interface moves slower than the phase front then between these two there is enough room for creation of kinks in the way described by the

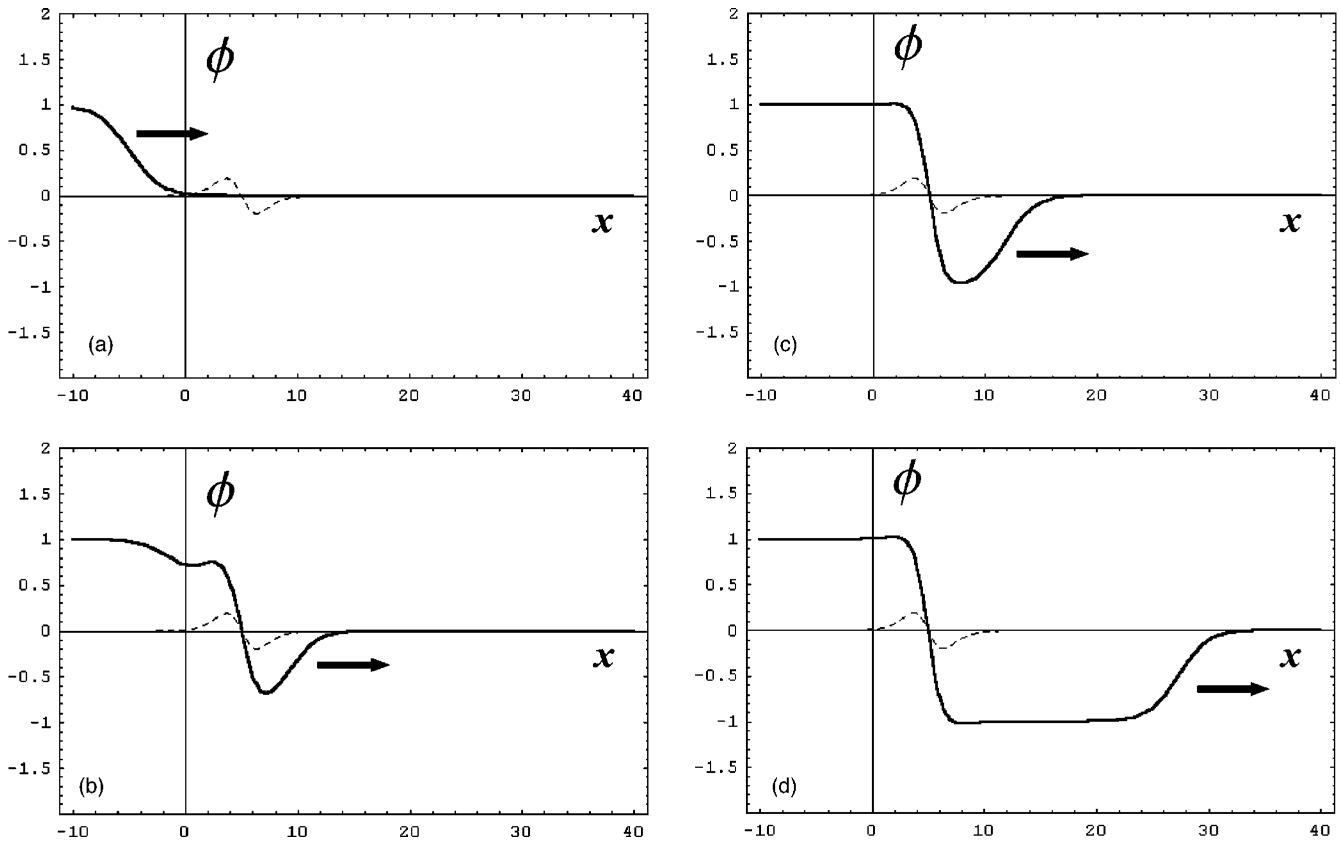

FIG. 2. At the beginning, the half kink moves in the direction of the impurity (a). If the orientation of the force distribution is negative and its magnitude is not too small, the interaction of the impurity with decaying false vacuum (b),(c) results in reversing the half kink into an inverse half kink moving in the direction of the x axis and production of the squeezed antikink at the position of the impurity center. The parameters are chosen as follows: $\mathcal{A}=-0.5,\Gamma=10,\lambda=1,a=1,x_0=5$. The impurity is represented by the dashed line.

Kibble-Zurek mechanism. In the opposite case the system left behind the phase front is free of defects.

In the present paper we show that in the system populated by impurities, creation of defects is possible even in the second case. Almost all defects created in these settings are located in the vicinity of the impurities. Let us add the term $\mathcal{D}(t,x)$ which describes some deterministic force distribution to Eq. (1):

$$\frac{1}{c^2}\partial_t^2\phi(t,x)+\Gamma\partial_t\phi(t,x)$$

$$=\partial_x^2\phi(t,x)+a\phi(t,x)-\lambda\phi^3(t,x)+\mathcal{D}(t,x). \quad (5)$$

In our numerical studies we choose a force distribution of the form

$$\mathcal{D}(x)=\pm\mathcal{A}\left(\frac{a}{\lambda}\right)^{3/2}\frac{\sinh\beta(x-x_0)}{\cosh^3\beta(x-x_0)}, \quad (6)$$

where $\mathcal{A}\in[0,\infty)$ describes the strength of the impurity force, $\beta\equiv\sqrt{(a/2)}b$ and $b\equiv\sqrt{1+\mathcal{A}/\lambda}$. This particular choice of function $\mathcal{D}(t,x)$ is motivated by the fact that for this choice we know an exact solution which has the form of a squeezed kink confined by the impurity center [6]. We considered the effect of propagation of the half kink, which is interface between false and true vacua, in the presumed force distribution.

We have found three qualitative different scenarios of the evolution of the field configuration. If the modulus of the amplitude of the force distribution $|\mathcal{A}|$ is sufficiently small then, after the period of the interaction of the half kink with the impurity center, we obtain half-kink moving in the initial direction. The evolution of the system is presented in Figs. 1(a)– 1(d). In this case the impurity is too weak to produce any additional kink structures in its vicinity. In this scenario, similar to the homogenous case, the system left behind the decaying false vacuum front is free of additional kink structures.

The situation changes significantly if the orientation of the force distribution changes and its magnitude is not too small, i.e., if parameter $\mathcal{A}$ became negative and its value became significant. In this case the interaction of the impurity with a decaying false vacuum results in a reversal of the half kink into inverse half kink and production of the squeezed antikink at the position of the impurity center. The process of the formation of this additional kink structure and the free propagation of the inverse half kink in the direction of the x axis is illustrated in Figs. 2(a)– 2(d).

The last possibility occurs for positively oriented and sufficiently strong impurities, i.e., for sufficiently large and

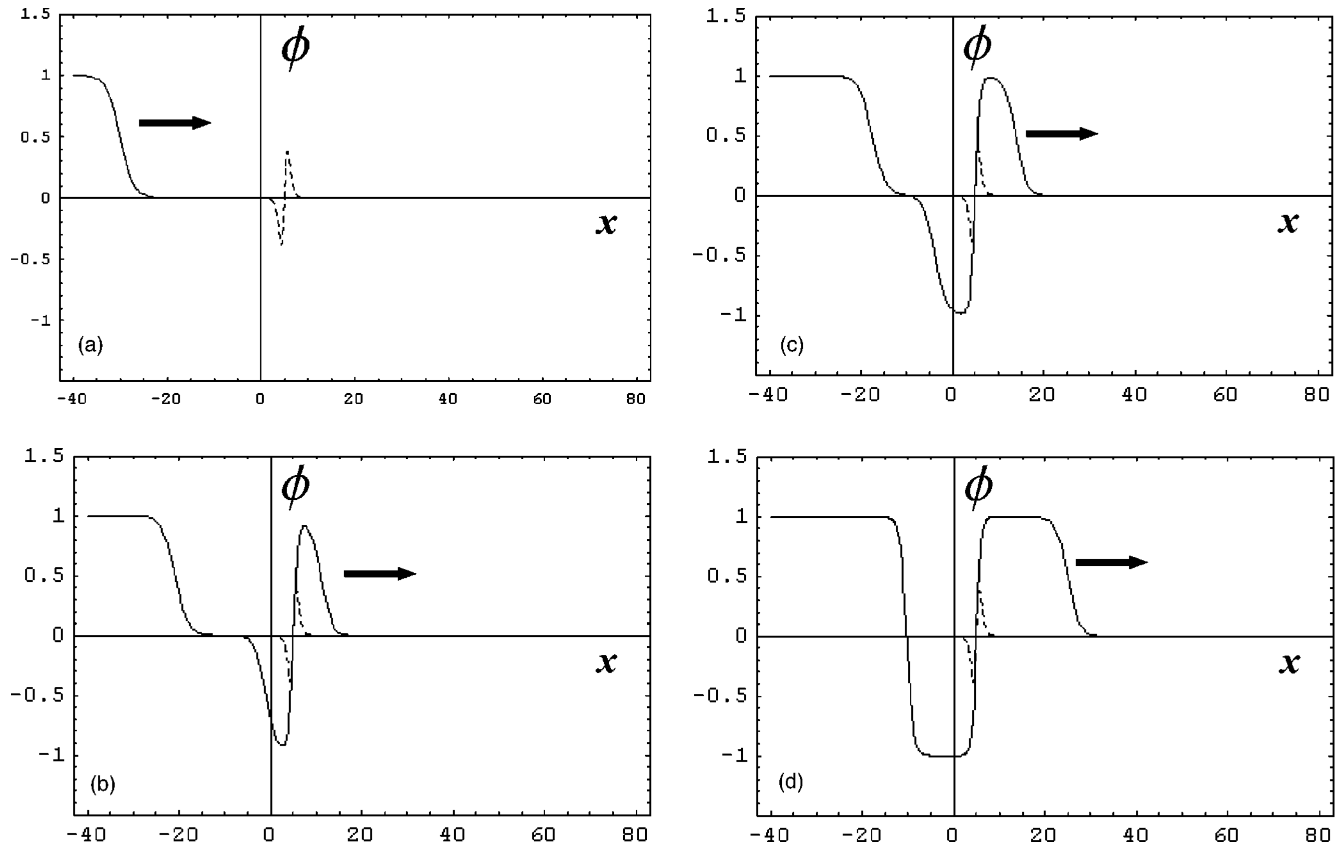

FIG. 3. First the half kink moves toward the imperfection positioned at x_0 (a) and then we observe formation of half kinks and inverse half kinks, and their movement in the opposite directions (b),(c). They meet each other and form free antikinks in addition to the existing squeezed kink formed at the position of the impurity (d). The parameters are chosen as follows: $\mathcal{A}=1$, $\Gamma=10$, $\lambda=1$, $a=1$, $x_0=5$. The dashed line represents the force coming from the impurity.

positive parameter $\mathcal{A}$. The evolution of the field configuration in this case is presented in Figs. 3(a)– 3(d). In Fig. 3(a) we have the half kink moving in the direction of the x axis. Then we observe the formation of half kink's and inverse half kink's and their movement in opposite directions. They meet each other and form a free antikink in addition to the existing squeezed kink formed at the position of the impurity. This description of the evolution of the considered field configuration follows from the arguments presented below.

In order to understand the meaning of the data presented in Fig. 3 let us consider the overdamped limit ($c\rightarrow\infty$) of the model (1), i.e.,

$$\Gamma\,\partial_t\phi(t,x)=\partial_x^2\phi(t,x)+a\,\phi(t,x)-\lambda\,\phi^3(t,x). \qquad (7)$$

First we consider an ansatz

$$\phi_D(t,x)=-\sqrt{\frac{a}{\lambda}}\;\frac{e^{\beta(x-d)}-e^{-\beta(x-d)}}{e^{\beta(x-d)}+e^{-\beta(x-d)}+f(t)}. \qquad (8)$$

In the considered limit we obtain an exact equation for test function f:

$$-\Gamma\dot{f}(t)=\frac{3}{2}af(t), \qquad (9)$$

which has two solutions. Trivial solution $f(t)=0$ leads to the known function $\phi_D(t,x)=\phi_{A\,free}(x-d)$. The second solution $f(t)=\exp[3a(t+t_0)/2\Gamma]=\exp[-\sqrt{a/2}v_s(t+t_0)]$ is much more interesting because it leads to a new solution of Eq. (7). It is worth stressing that at first sight this new solution looks similar to the sum of half kinks moving in opposite directions but this is not the case, i.e., $\phi_D(t,x)\neq\phi_H^{(+)}(x-d-v_st)+\phi_H^{(-)}(x-d+v_st)$. Let us notice that for the sum of half kinks, function f has the form $f(t)=2\cosh[\sqrt{a/2}v_s(t+t_0)]$.

We showed that half-kink-like solutions meet at some position preceding the localization of the squeezed kink trapped by the impurity center. The field configuration, which is the final state for late times, can be approximately described as a sum of the squeezed kink [6] confined by the impurity center, the free antikink associated with the squeezed kink, and the half kink propagating freely to infinity. If we choose an ansatz

$$\phi(t,x)=\phi_{K\,sq}(x-x_0)+\phi_{A\,free}(x-d)+\phi_H(x-v_st) \qquad (10)$$

with unknown antikink position d, then equation of motion (5) can be reduced to the following algebraic constraint on parameter d:

$$(\phi_{K\ sq}+\phi_{A\ free})[\phi_{K\ sq}\phi_{A\ free}+\phi_H(\phi_{K\ sq}+\phi_{A\ free}+\phi_H)]=0. \tag{11}$$

Let us rescale fields $\phi\equiv\sqrt{a/\lambda}\ F$ and rewrite the last equation with the use of the rescaled variable F:

$$(F_{Ksq}+F_{A\ free})[F_{Ksq}F_{A\ free}+F_H(F_{Ksq}+F_{A\ free}+F_H)]=0. \tag{12}$$

This equation is approximately fulfilled almost everywhere. The disappearing of the left side of this equation in the neighborhood of the antikink localization can be achieved by the proper choice of the parameter d. Let us notice that in the vicinity of the antikink localization $F_{A\ free}|_{x\approx d}\approx 0$ and $F_H|_{x\approx d}\approx 1$. On the other hand, if we consider regime $\sqrt{a/2}\ b(d-x_0)\ll 1$ then $F_{K\ sq}|_{x\approx d}\approx F_{K\ sq}(d-x_0)\approx\sqrt{a/2}\ b(d-x_0)$ and we can rewrite Eq. (12) in the approximate form

$$\sqrt{\frac{a}{2}}b(d-x_0)\left[\sqrt{\frac{a}{2}}b(d-x_0)+1\right]=0.$$

It is worth stressing that although the localization of the antikink associated with the squeezed kink is not determined very precisely, $d\approx x_0-\sqrt{2/a}/b$, it is uniquely defined by the impurity position x_0 and its strength $\mathcal{A}$.

Now the interpretation of the results presented in Fig. 3 (at least in the overdamped limit) is straightforward. We can consider the profile presented in Fig. 3 as a sum of the half kink moving in the direction of the x axis, the squeezed kink located at the impurity center, and the accompanying "double half kink" ϕ_D evolving to the free antikink configuration. In all simulations we assumed large damping which prevents the creation of additional unstable excitations (all these excitations decay for sufficiently late time).

Finally we also show the evolution of the system in the presence of a few defects in Figs. 4(a),4(b). Numerical simulations show that for the considered form of the force distribution each impurity center is occupied by the squeezed kink and accompanied by the free antikink. It is easy to imagine that if the force distribution is more complicated and to some degree random then under a fixed asymptotic the field configuration in the region of the nonzero force distribution may have a complicated structure, however, this structure fits the boundary conditions. Defects in this situation can be identified as points where the scalar field changes its sign. The stable configuration in this system is achieved after sufficiently long time due to the decay of the initial state by radiation and damping of all excitations present in the system.

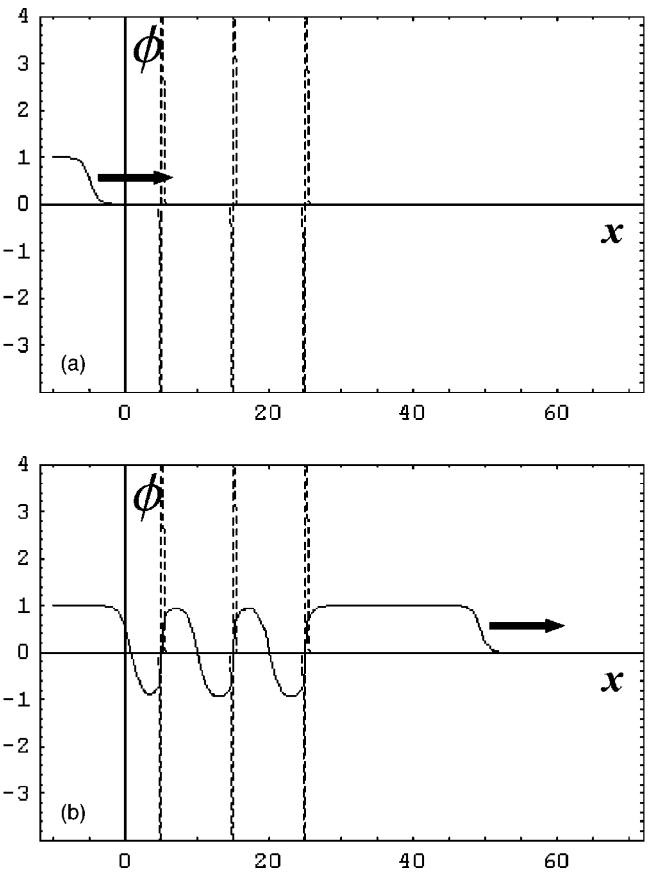

FIG. 4. The evolution of the system in the presence of a few defects. Numerical simulations show that for very strong force distribution each impurity center is occupied by the squeezed kink and accompanied by the free antikink (b). The parameters are chosen as follows: $\mathcal{A}=199$, $\Gamma=1$, $\lambda=1$, $a=1$, $x_{01}=5$,$x_{02}=15$, $x_{03}=25$.

III. REMARKS

The context of our investigations is composed of the studies of the creation of defects behind the shock wave quench front $a=a(x-t/v)$. In fact in our studies we do not change a but, at any rate, we can draw some conclusions concerning this case. We know that in homogenous media there are two different regimes of creation defects behind the quench front. In the first regime the speed of propagation of the quench front v is lower than the speed of propagation of the front of the decaying false vacuum v_s. In this regime the half kink moves in step with the quench front and therefore the choice of vacuum is solely determined by the choice of boundary conditions. Due to the stability of this solution, no kinks can be created behind the quench front. In the second regime v_s is smaller than the speed of propagation of the quench front v and therefore there is enough room for instability between the quench front and the front of the decaying false vacuum leading to the production of defects.

In the present paper we concentrated on creation of defects behind the quench front in the first regime, i.e., in the regime where in homogenous media the system left behind this slow front is free of defects. We showed that even in this regime some number of defects behind the front is created in inhomogeneous media. This number and the particular form of final configuration is determined by the force distribution and the boundary conditions. We showed for a particular form of the impurity force distribution, the final configurations which posess additional kink structures created behind the decaying false vacuum front. We would like to stress that in this regime creation of kink structures is suppressed in the homogenous case. Although the particular form of the assumed force distribution may seem to be a little artifi-

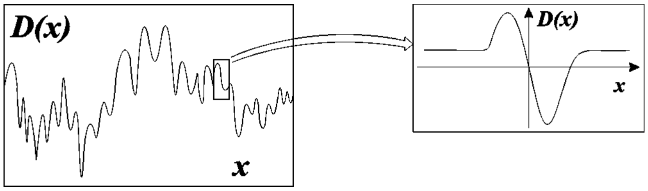

FIG. 5. Any nonconstant force distribution has locally decreasing-increasing (increasing-decreasing) character.

cial we would like to stress that its crucial feature is the decreasing-increasing (increasing-decreasing) character which is a generic behavior of any nonconstant force distribution (see Fig. 5).

It seems that in an inhomogeneous medium there are two components of the number density of produced defects. One part of defects is produced in the neighborhoods of the impurities and they are persistently confined by the impurities. The second component consists of free defects produced in homogenous areas located between impurities. This component is present in the second regime mentioned above. Moreover this component decreases with time as a consequence of the kink-antikink annihilation. Creation of kink-antikink pairs is also possible but at this stage of evolution it is much less efficient than annihilation. The contribution of each component to the total number density, depends on the separation of the impurities and on their strength. For instance, if separation of the strong impurities is comparable with correlation length (or smaller) then there is no room for creation of free defects. In the opposite regime, i.e., for widely separated impurities, the main part of the kinks is produced in the areas located between impurity centers and therefore the total number density is dominated by free kinks [5,6].

ACKNOWLEDGMENTS

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